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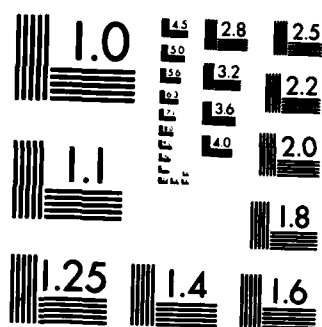
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PRELIMINARY DEVELOPMENT OF AN
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SUPERCRITICAL WING DESIGN
OPTIMIZATION APPLICATIONS

by

Stephen S. Stahara

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Stephen S. Stahara

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PRELIMINARY DEVELOPMENT OF AN APPROXIMATION
PROCEDURE FOR SUPERCRITICAL WING DESIGN
OPTIMIZATION APPLICATIONS

Stephen S. Stahara

Summary

An investigation was carried out involving the preliminary development of an approximation procedure and associated computational codes for rapidly determining approximations to nonlinear, three-dimensional flow solutions, with the purpose of establishing a method for minimizing the computational work requirements associated with design optimization studies of supercritical wings. The results ~~reported here~~ concern the extension of a previously-developed successful approximation method for determining accurate approximations to two-dimensional nonlinear transonic flows involving the simultaneous change of multiple geometric and/or aerodynamic parameters. The specific development involves combination of the nonlinear approximation procedure with the FLO22 three-dimensional wing transonic flow solver together with the CONMIN optimization ~~procedure~~ in a configuration suitable for supercritical wing design/optimization studies.

(program

Key words -

1. INTRODUCTION

The remarkable success of advanced computational methods for determining accurate solutions to increasingly complex fluid dynamic phenomena has now been well established across a broad range of flow problems. What has also become simultaneously apparent with this success is that a major impediment exists to the implementation of these emerging codes in highly-repetitive usage applications. This is due to the excessive computational demands required by their straightforward application. Many such applications simply cannot afford the computational cost associated with the repetitive use of these higher-level numerical solvers. Thus a need clearly exists for the development and implementation of sufficiently general and accurate nonlinear approximation methods that are capable of reducing these computational requirements. While this need exists across a spectrum of aerodynamic uses, it is particularly high in supercritical wing optimization applications. For that application, both the basic aerodynamic computation is highly time consuming and the number of design variables usually required for a satisfactory result is large, resulting in any optimization study becoming computationally expensive under the best of circumstances, and in many instances prohibitively so.

The final or ultimate goal beyond this preliminary study is to develop and demonstrate the means for substantially reducing the overall computational requirements necessary for general supercritical wing design optimization. It is conceived that these methods would be coupled with high run-time general supercritical wing computational flow field solvers and would be used in conjunction with them in applications where large numbers of related nonlinear solutions are needed. The time saving would be accomplished by development of rapid approximation methods that would enable the actual number of expensive numerical flow solutions required in any optimization study to be reduced to a minimum. The actual implementation would entail using the rapid approximation method together with a certain minimum number of expensive flow solutions to then subsequently predict all of the aerodynamic flow solutions required by the optimization search process as that procedure searches through the design variable solution space to reach the optimum design.

That such procedures are achievable has now been successfully demonstrated for two-dimensional flows. In studies made by the present author and reported in Refs. 1-7, a remarkable nonlinear approximation method was developed and extensively tested on a wide range of both continuous and discontinuous nonlinear flow problems. Its ability to accurately predict nonlinear solutions of primary interest to this study was first confirmed in case studies involving a variety of strongly nonlinear transonic flows. The method was then coupled with an optimization procedure and tested on several two-dimensional design

problems. The results demonstrated the potential of the approximation method to reduce the computational work in such applications by an order in magnitude with no degradation in accuracy.

The work reported here involves the preliminary extension of these methods and concepts to the three-dimensional supercritical wing optimization design problem. The specific implementation involves development of the nonlinear approximation method in a form suitable for predicting surface properties on three-dimensional supercritical wings; and then integration of that form of the approximation method with a wing design optimization procedure. The FLO22OPT wing optimization procedure recently developed at Ames Research Center was selected for this study. That procedure consists of the CONMIN optimization code (Ref. 8) coupled with the FLO22 three-dimensional full potential solver (Ref. 9) for determining transonic flows past wing-body combinations.

2. ANALYSIS

2.1 DESCRIPTION OF THE NONLINEAR APPROXIMATION CONCEPT

The classical approach of developing a perturbation or approximation analysis -- that is, by establishing and solving a series of linear perturbation equations in the manner of Van Dyke (Ref. 10) -- appears as an obvious choice for the current application. However, results from the work reported in Ref. 1 demonstrate that for applications to sensitive flows such as occur in most transonic situations, the basic linear variation assumption fundamental to such a technique is sufficiently restrictive that the permissible range of parameter variation is so small to be of little practical use. An interesting and novel alternative to the linear perturbation equation approach has recently been successfully examined in which a correction or approximation technique is used that employs two or more nonlinear base solutions. For the approximation method, the basic perturbation solution is determined simply by differencing two nonlinear flow solutions removed from one another by some nominal change of a particular flow or geometrical quantity. A unit perturbation solution is then obtained by dividing that result by the change in the perturbed quantity. Related solutions are determined by multiplying the unit perturbation by the desired parameter change and adding that result to the base flow solution. This simple procedure, however, only works directly for continuous flows for which the perturbation change does not alter the solution domain. For those perturbations which change the flow domain, coordinate stretching is necessary to ensure proper definition of the unit perturbation solution. Similarly, for discontinuous flows, coordinate straining is necessary to account for movement of discontinuities due to the perturbation. We will discuss in detail the importance of coordinate straining to the approximation method below.

The attractiveness of such an approximation method is that it is not restricted to a linear variation range but rather replaces the nonlinear variation between two base solutions with a linear fit. This de-emphasizes the dependence and sensitivity inherent in the linear perturbation equation method on the local rate of change of the base flow solution with respect to the varied quantity. For many applications, particularly at transonic speeds, the flow is highly sensitive, and the linear range of parameter variation can be sufficiently small to be of no practical use. Furthermore, other than the approximation of a linear fit between two nonlinear base solutions, this new method is not restricted by any further approximations with respect to the governing differential equations and boundary conditions. Rather, it retains the full character of the original methods used to calculate the base flow solutions. Most importantly, no perturbation differential equations have to be posed and solved, only algebraic ones. In fact, it isn't even necessary to know the exact form of the perturbation equation, only that it can be obtained by some systematic

procedure and that the perturbations thus defined will behave in some 'generally appropriate' fashion so as to permit a logical perturbation analysis. For situations involving perturbations of physical parameters, such as reported here, the governing perturbation equations are usually transparent, or at least readily derivable. Finally, in applying this method it isn't necessary to work with primitive variables; rather the procedure can be applied directly to the final quantity desired. An important qualification of this method is that the two base solutions required for each parameter perturbation considered must be topologically similar, i.e., discontinuities or other characteristic features must be present in both base solutions used to establish the unit perturbation.

The fundamental idea underlying coordinate straining as it relates to the application of perturbation methods to nonlinear flows is illustrated geometrically in Figure 1. In the upper plot on the left, two typical transonic pressure distributions are shown for a highly supercritical flow about a nonlifting symmetric profile. The distributions can be regarded as related nonlinear flow solutions separated by a nominal change in some geometric or flow parameter. The shaded area between the solutions represents the perturbation result that would be obtained by directly differencing the two solutions. We observe that the perturbation so obtained is small everywhere except in the region between the two shock waves, where it is fully as large as the base solutions themselves. This clearly invalidates the perturbation technique in that region and most probably somewhat ahead and behind it as well. The key idea of a procedure for correcting this, pointed out by Nixon (Refs. 11 and 12), is first to strain the coordinates of one of the two solutions in such a fashion that the shock waves align, as shown in the upper plot on the right of Figure 1, and then determine the unit perturbation. Equivalently, this can be considered as maintaining the shock wave location invariant during the perturbation process, and assures that the unit perturbation remains small both at and in the vicinity of the shock wave. Obviously, shock points are only one of a number of characteristic high-gradient locations such as stagnation points, maximum suction pressure points, etc., in which the accuracy of the perturbation solution can degrade rapidly. The plots in the lower left part of the Figure 1 indicate such a situation and display typical transonic pressure distributions which contain multiple shocks and high-gradient regions. Simultaneously straining at all these locations, as indicated in the lower right plot, serves to minimize the unit perturbation over the entire domain considered, and provides the key to maximizing the range of validity of the perturbation method.

2.2 PREVIOUS APPLICATIONS

At this point, the approximation concept based on the ideas discussed above has both been implemented and thoroughly tested in a wide range of problems. In Ref. 1, several candidate approximation methods were studied and the most promising method was identified. Extensive development and testing of that method was then carried out in

Ref. 4 on a large number of nonlinear flow problems involving single-parameter changes of a variety of flow and geometric parameters. Subcritical and supercritical flows past isolated airfoils and compressor cascades were considered, with particular emphasis placed on supercritical transonic flows which exhibited large surface shock movements over the parametric range studied. Comparisons of the approximation predictions with the corresponding 'exact' nonlinear solutions indicated a remarkable accuracy and range of validity of the approximation method. For example, Figure 2 from Ref. 4 provides a comparison of results illustrating the remarkable ability of the approximation method to predict nonlinear supercritical transonic flows. These results are for surface pressures obtained from full potential solutions and represent nonlifting flows past several NASA four-digit, thickness-only airfoils at $M_\infty = 0.820$. The results indicated by the dotted and dashed lines were obtained for thickness ratios of $\tau = 0.12$ and 0.08 , respectively. Those results were used to define the unit perturbation required by the approximation method. With that unit perturbation in hand, the approximation method was then employed to predict surface pressure results for thickness ratios $\tau = (0.110, 0.105, 0.100, 0.095)$. The approximation results, indicated by the open symbols, were then compared with full nonlinear results obtained by running the full potential solver at those thickness ratios. As can be seen, the results are essentially identical, in particular, in the region of the strong shock.

The approximation method was next extended (Ref. 6) to treat simultaneous multiple-parameter perturbations. Extensive testing of the method demonstrated remarkable accuracy and range of validity of the multiple-parameter approximation procedure in direct correspondence with the previous results obtained for single-parameter changes. Additionally, initial applications of the multiple-parameter approximation method combined with an optimization procedure were also made to several two-dimensional airfoil design problems. The results demonstrated the potential of the approximation method for reducing the computational work in certain applications by an order of magnitude with no degradation in accuracy. Finally, in Ref. 7, the approximation method, configured in a form suitable for predicting an arbitrary number of simultaneous multiple parameter changes, was combined with the COPES/CONMIN optimization driver (Ref. 13) and coupled with the NASA TSONIC full potential blade-to-blade turbomachinery solver (Ref. 14). A series of calculations of the combined code, named BLDOPT, have verified the procedure, demonstrated the accuracy of the approximation-predicted results, and established benchmark guidelines of the potential for computational savings of the method under the various user options included in the code. In general, the approximation method was found to be capable of providing an order of magnitude reduction in computational work in those applications which involved essentially subcritical or weakly supercritical turbomachinery flows.

2.3 THEORETICAL FORMULATION: APPROXIMATION PREDICTION OF SURFACE PROPERTIES ON SUPERCRITICAL WINGS

The underlying reason of the remarkable accuracy of the approximation method developed in this study lies in the use of coordinate straining to define the unit perturbation. As shown in Figure 1, where the perturbation between two nonlinear solution states is displayed graphically as the shaded area between the base and the strained and unstrained calibration solution, coordinate straining provides the ability to account accurately for the displacement of a multiple number of discontinuities and maxima of high-gradient regions due to a parameter change. This enables the perturbation method to maintain very high accuracy in regions of high gradients where most perturbation methods commonly fail, and to maintain that accuracy over large parametric ranges.

In what follows, we provide a brief account of the theoretical essentials of the strained-coordinate perturbation concept as configured and implemented in the present design application. This is to predict simultaneous multiple-parameter perturbation flow solutions for surface properties of supercritical wings for use in optimized wing design. The flow solutions thus considered can contain a total number N of discontinuities or high-gradient continuous regions.

To proceed with the theoretical basis of the approximation method as applied to simultaneous multiple-parameter perturbations of flows containing multiple shocks or high-gradient regions, consider the formulation of the procedure at the full potential equation level, since all of the results presented here are based on that level. Denote the operator L acting on the full velocity potential ϕ as that which results in the three dimensional full-potential equation for ϕ , i.e.,

$$L[\phi] = 0 \quad (1)$$

If we now expand the potential in terms of zero and higher-order components in order to account for the variation of M arbitrary geometrical or flow parameters q_j from their base flow values q_{0j}

$$\phi = \phi_0 + \sum_{j=1}^M \epsilon_j \phi_{1j} + \dots \quad (2)$$

$$q_j = q_{0j} + \Delta q_j$$

and then insert these expansions into the governing Equation (1), expand the result, order the equations into zero and first-order components, and make the obvious choice of expansion parameters $\epsilon_j = \Delta q_j$ we obtain the following governing equations for the zero and M first-order components

$$L[\Phi_0] = 0$$

(3)

$$L_1[\Phi_{1j}] + \frac{\partial}{\partial q_j} L[\Phi_0] = 0$$

Here L_1 is a linear operator whose coefficients depend on zero-order quantities and $\partial L[\Phi_0]/\partial q_j$ represents a 'forcing' term due to the q_j^{th} perturbation. Actual forms of L_1 and the 'forcing' term are provided in Ref. 1 for a variety of flow and geometry parameter perturbations of a two-dimensional turbomachine, and in Ref. 15 for profile shape perturbations of an isolated airfoil. An important point regarding Equation (3) for the first-order perturbations is that these equations represent a unit perturbation independent of the actual value of the perturbation quantity ϵ_j .

Appropriate account of the movement of a multiple number of discontinuities and maxima of high-gradient regions due to the changes in the parameters q_j is now accomplished by the introduction of strained coordinates (s,t) in the form

$$\begin{aligned} x &= s + \sum_{j=1}^M \epsilon_j x_1(s,t) \\ y &= t + \sum_{j=1}^M \epsilon_j y_1(s,t) \end{aligned} \quad (4)$$

where

$$\begin{aligned} x_1(s,t) &= \sum_{i=1}^N \delta x_i(t) x_{1i}(s,t) \\ y_1(s,t) &= \sum_{i=1}^N \delta y_i(t) y_{1i}(s,t) \end{aligned} \quad (5)$$

and $\epsilon_j \delta x_i$, $\epsilon_j \delta y_i$ represent individual x and y displacements due to perturbation of the q_j^{th} parameter of the N strained points, and $x_{1i}(s,t)$, $y_{1i}(s,t)$ are straining functions associated with each of the N strained points. For the applications considered here, we have assumed that all discontinuities such as shock waves or other high-gradient region maxima occur essentially normal to the wing planform so that only the (x,y) coordinates require straining. This simplification is not strictly necessary and could be relaxed in future applications. However, the effect of this assumption on the prediction of surface properties via the approximation method is known from extensive studies of the two-dimensional case to be of higher order for most optimized design flow situations of aerodynamic interest. Introducing the strained coordinate Equations (4) and (5) into the expansion formulation leaves the zero-order result in Equation (3) unchanged, but results in a

change of the following form for the j^{th} perturbation

$$L_1[\phi_{1j}] + L_{2j}[\phi_0] + \frac{\partial}{\partial q_j} L[\phi_0] = 0 \quad (6)$$

Here the operators are understood to be expressed in terms of the strained (s, t) coordinates, and the additional operator L_{2j} arises specifically from displacement of the strained points. In Refs. 12 and 15, specific expressions for L_{2j} are provided for selected perturbations involving transonic small-disturbance and full-potential equation formulations. The essential point, however, with regard to perturbation Equation (6) expressed in strained coordinates is that it remains valid as before for a unit perturbation and independent of ϵ_j .

In employing the approximation method, Equation (6) for the j^{th} unit perturbation is solved by taking the difference between two solutions obtained by the full nonlinear procedure after appropriately straining the coordinates. If we designate the solutions for some arbitrary dependent flow quantity Q as base Q_0 and calibration Q_{cj} , respectively, of the varied independent parameter q_j , we have for the predicted flow at some new parameter value q_j for all the M parameters

$$Q(x, y) = Q_0(s, t) + \sum_{j=1}^M \epsilon_j Q_{1j}(s, t) + \dots \quad (7)$$

where

$$Q_{1j}(s, t) = \frac{Q_{cj}(\bar{x}_j, \bar{y}_j) - Q(s, t)}{\epsilon_j} \quad (8)$$

$$\bar{x}_j(t) = s + \sum_{i=1}^N \bar{\epsilon}_j \delta x_i(t) x_{1i}(s, t) \quad (9)$$

$$\bar{y}_j(t) = t + \sum_{i=1}^N \bar{\epsilon}_j \delta y_i y_{1i}(s, t) \quad (10)$$

$$x = s + \sum_{j=1}^M \frac{\epsilon_j}{\bar{\epsilon}_j} (\bar{x}_j(t) - s) \quad (11)$$

$$y = t + \sum_{j=1}^M \frac{\epsilon_j}{\bar{\epsilon}_j} (\bar{y}_j(t) - t) \quad (12)$$

$$\bar{\epsilon}_j = q_{cj} - q_{0j} \quad (13)$$

$$\epsilon_j = q_j - q_{0j} \quad (14)$$

We note that in order to determine the first-order corrections $Q_{1j}(s,t)$, we require one base and M calibration solutions in which the calibration solutions are determined by varying each of the M arbitrary independent parameters q_j by some nominal amount from the base flow value while keeping the others fixed at their base values. In this way, the first-order corrections Q_{1j} can be determined from Equation (8) where Q_{cj} is defined as the calibration solution corresponding to changing the j^{th} parameter to a new value q_{cj} , \bar{x}_j is the strained coordinate pertaining to the Q_{cj} calibration solution, and $\bar{\epsilon}_j = q_{cj} - q_{0j}$ represents the change in the q_j parameter from its base flow value. Thus,

$$\bar{\epsilon}_j \delta x_i(t) = (x_i^c(t) - x_i^0(t))_j \quad (15)$$

$$\epsilon_j \delta x_i(t) = \frac{\epsilon_j}{\bar{\epsilon}_j} (x_i^c(t) - x_i^0(t))_j \quad (16)$$

$$\bar{\epsilon}_j \delta y_i = (y_{Ti}^c - y_{Ti}^0)_j \quad (17)$$

$$\epsilon_j \delta y_i = \frac{\epsilon_j}{\bar{\epsilon}_j} (y_{Ti}^c - y_{Ti}^0)_j \quad (18)$$

where $\epsilon_j \delta x_i(t)$ given in Equation (15) represents the x displacement of the i^{th} invariant line at the spanwise t location in the j^{th} calibration solution from its base flow location due to the selected change $\bar{\epsilon}_j$ in the q_j parameter given by Equation (13), $\epsilon_j \delta x_i(t)$ given in Equation (16) represents the predicted x displacement of the i^{th} invariant line at the spanwise t from its base flow location due to the desired change ϵ_j in the q_j parameter given by Equation (14), $\bar{\epsilon}_j \delta y_i$ given in Equation (17) represents the y displacement of the tip of the i^{th} invariant line in the j^{th} calibration solution from its base flow location due to the selected change $\bar{\epsilon}_j$ in the q_j parameter given by Equation (13), and $\epsilon_j \delta y_i$ given in Equation (18) represents the predicted y displacement of the tip of the i^{th} invariant line from its base flow location due to the desired change ϵ_j in the q_j parameter given by Equation (14), $x_{1j}(s,t)$ is a unit-order straining function having the property that

$$x_{1i}(x_k^0(t), t) = \begin{cases} 1 & k = i \\ 0 & k \neq i \end{cases} \quad (17)$$

which assures alignment of the i^{th} invariant line at the t spanwise location between the base and calibration solutions, and $y_{1i}(t)$ is a unit-order straining function having the property that

$$y_{1i}(y_{T_k}^0) = \begin{cases} 1 & k = i \\ 0 & k \neq i \end{cases} \quad (18)$$

which assures alignment of the tip of the i^{th} invariant line between the base and calibration solutions.

In addition to the single conditions given by Equations (17) and (18) on the straining functions, it may be convenient or necessary to impose additional conditions at other locations along the contour. For example, it is usually necessary to hold invariant the end points along the contour, as well as to require that the straining vanish in a particular fashion in those locations. All of these conditions, however, do not serve to determine the straining uniquely. The nonuniqueness of the straining, nevertheless, can often be turned to advantage, either by selecting particularly simple classes of straining functions or by requiring the straining to satisfy further constraints convenient for a particular application.

The fact of nonuniqueness of straining function, however, raises a further question of the dependence of the final approximation-predicted result on choice of straining function. An initial example of the effect of employing two different straining functions for a strongly supercritical two-dimensional flow was provided in Ref. 12, and in Ref. 4 a detailed examination was made of the dependence of approximation results on several classes of different straining functions. Although it can be demonstrated (Ref. 3) that the final approximation-predicted result obtained when employing strained coordinates is formally independent of the particular straining function used -- provided that the straining function moves the invariant points to the proper locations -- the results of Ref. 4 demonstrate that, under certain conditions, particular classes of straining functions can induce spurious approximation results. The underlying reason is that, while the approximation-predicted results at and in the vicinity of invariant points are independent of the choice of straining function (provided invariant point locations are preserved), some classes of straining functions have the undesirable property of producing unwanted straining in certain regions removed from the invariant points. The correction for this deficiency, which was found in Ref. 4 and has proven effective in all case studies undertaken, is to employ linear piecewise-continuous straining functions. This both preserves the accuracy of the

approximation results in the vicinity of the invariant points, and introduces no excessive straining in regions removed from those locations.

For linear piecewise-continuous straining functions, the functional forms of the straining can be compactly written. For the x displacement we have

$$\bar{x}_j(t) = s + \left\{ \frac{x_{i+1}^0(t) - s}{x_{i+1}^0 - x_i^0} \cdot (x_i^C(t) - x_i^0(t))_j + \frac{s - x_i^0(t)}{x_{i+1}^0(t) - x_i^0(t)} \cdot (x_{i+1}^C(t) - x_{i+1}^0(t))_j \right\} H(x_{i+1}^0(t) - s) \cdot H(s - x_i^0(t)) \quad (19)$$

where H denotes the Heaviside step function. As discussed above, it is usually necessary to hold invariant both of the end points along the contour in addition to the points corresponding to discontinuities or high-gradient maxima. Consequently, for the results reported here, the array of x invariant points in the base and calibration solutions have been taken as

$$x_i^0(t) = \{0, x_1^0(t), x_2^0(t), \dots, x_n^0(t), 1\} \quad (20)$$

$$x_{ij}^C(t) = \{0, x_{1j}^C(t), x_{2j}^C(t), \dots, x_{nj}^C(t), 1\}$$

where the contour length at the spanwise location t has been normalized to unity and where n is the number of invariant points along the contour exclusive of the end points.

Similarly, for the y displacements of the tips of the discontinuity lines we have

$$\bar{y}_j(t) = t + \left\{ \frac{y_{T_{i+1}}^0 - t}{y_{T_{i+1}}^0 - y_{T_i}^0} \cdot (y_{T_i}^C - y_{T_i}^0)_j + \frac{t - y_{T_i}^0}{y_{T_{i+1}}^0 - y_{T_i}^0} \cdot (y_{T_{i+1}}^C - y_{T_{i+1}}^0)_j \right\} H(y_{T_{i+1}}^0 - t) \cdot H(t - y_{T_i}^0) \quad (21)$$

where the spanwise locations of the tips of the discontinuity lines in the base and calibration solutions have been taken as

$$\begin{aligned} y_{T_i} &= \{y_{T_1}, y_{T_2}, y_{T_3}, \dots, y_{T_n}\} \\ y_{T_{ij}}^c &= \{y_{T_1}^c, y_{T_2}^c, y_{T_3}^c, \dots, y_{T_n}^c\}_j \end{aligned} \tag{22}$$

3. RESULTS

Because the ultimate utility of the approximation methods being developed in this preliminary investigation lies in engineering design of optimized supercritical wings, the primary objective of the current study was to examine the accuracy and range of validity of the approximation method in test cases characteristic of that environment. The objective here was to better understand the approximation method's present strengths and to uncover any of its weaknesses. Toward that end, we have tested the approximation method in a stand-alone mode in several supercritical wing case studies involving the simultaneous change of multiple parameters comprised of both flow and geometric quantities. As with the previous testing of the approximation method in two-dimensional applications, emphasis was placed on transonic flows that are strongly supercritical over the parametric range studied. This was done so as to provide as severe a test as possible. In addition to the stand-alone testing of the approximation method, the method was integrated with the FL022OPT using optimization procedure which consists of the FL022 three-dimensional full potential solver (Ref. 9) coupled with the CONMIN optimization code (Ref. 8). Next, several preliminary optimization case studies were carried out to examine the approximation method's ability to perform in an actual supercritical wing design environment.

3.1 APPROXIMATION METHOD PREDICTION OF SUPERCRITICAL WING PRESSURES

Several case studies were undertaken to examine the accuracy of the approximation method in predicting strongly supercritical transonic flows past wings. The particular wing geometry configurations selected for study, although not specifically directed toward a currently-operational design, were chosen to be representative of modern transport aircraft wing designs. Figure 3 displays the shape of the planform that was selected. The section profiles of the wing consisted of NACA 65A215 profiles, with the maximum thickness of the sections varying linearly across the span from $\tau = 0.08$ at the root chord to $\tau = 0.06$ at the tip.

In Figure 4, we present a comparison of approximation-predicted and exact nonlinear results for the simultaneous perturbation of oncoming Mach number and angle of attack of highly-supercritical flows past the wing shown in Figure 3. The base flow chosen for these results is at $M_\infty = 0.80$ and $\alpha = 0^\circ$, while the calibration solutions required to determine the unit perturbations in M_∞ and α were selected respectively at $(M_\infty, \alpha) = (0.82, 0^\circ)$ and $(0.80, 3^\circ)$. All of these flow field solutions, as well as those employed in the optimization studies presented below, were obtained from the FL022 code employing a medium density mesh with a total of 60 chordwise points, 11 spanwise, and 12 normal mesh planes. These flow solutions were then used together with the approximation method to predict the upper surface pressure distribution results at

$(M_\infty, \alpha) = (0.85, 1^\circ)$ displayed as the open circles in Figure 4 at the various spanwise locations indicated. Those results are meant to be compared with the corresponding "exact" nonlinear results which are indicated as the solid lines. In the approximation results, the leading and trailing edges, and the shock point were held invariant. The shock point locations for this example, as well as for all of the results presented here, were determined as the point where the pressure coefficient passed through critical with compressive gradient.

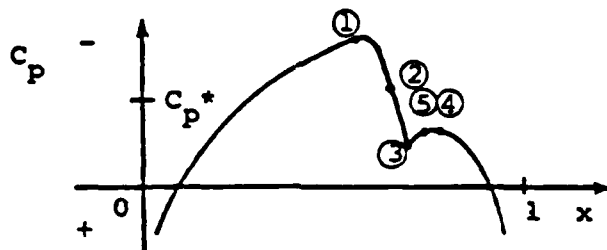
With regard to the results, we note that the comparison between the approximation-predicted and the exact nonlinear result is quite good, in particular in the region of the shock. The approximation method is able to predict accurately both shock location and the critical post-shock behavior. Results for the region from the leading edge to points ahead of the shock are essentially identical to the exact nonlinear solution, as are results aft of the post-shock region. We note that the particular parameter values of $(M_\infty, \alpha) = (0.85, 1^\circ)$ selected for the predicted solution represent reasonably substantial excursions from the base and calibration values. Nevertheless, the approximation method is able to predict simultaneous parameter variations over this range accurately.

In Figure 5 we present analogous results for a three-parameter perturbation of strongly supercritical full-potential flows past a similar wing as that shown in Figure 3, except that the wing tip chord ratio for the new wing was increased to 0.75 from 0.50. These results involved the simultaneous perturbation of oncoming Mach number, angle of attack, and wing thickness ratio. The base flow parameters involved an oncoming Mach number of $M_\infty = 0.80$, angle of attack $\alpha = 1^\circ$, and root chord thickness ratio $\tau = 0.70$. The three calibration solutions required to account for changes in the three varied parameters involved the following parameter value changes $(M_\infty, \alpha, \tau) = (0.82, 2^\circ, 0.85)$. For example, the calibration solution for Mach number was run at the new Mach number $M_\infty = 0.82$, with the other two parameters held fixed at their base flow values. Thus, the parameter values for the Mach number calibration solution were $(M_\infty, \alpha, \tau) = (0.82, 1^\circ, 0.70)$, with corresponding values for the other two calibration solutions. The comparison of the approximation-predicted and exact nonlinear results are for the parameter values of $(M_\infty, \alpha, \tau) = (0.85, 0^\circ, 0.80)$, and are shown at the various spanwise locations in Figure 5. This particular set of flows was again selected, as in the previous example, because of the presence of a strong shock across the span and a high sensitivity to parameter change. As with the previous results shown in Figure 4 for two simultaneous parameter variations, we observe that the approximation predictions are once again notably accurate for this three parameter perturbation. The approximation method is able both to track the location of the shock, as well as to predict the pressure characteristics in the pre-shock and post-shock regions. We note with regard to the shock topologies in these examples that both of the case studies presented involve full span shocks. Finally, we note that these results were obtained with the approximation method configured in a modified and improved form from that previously reported in References 1-7. This involved the development and incorporation of a more accurate

invariant point straining procedure than was heretofore used. This improvement utilizes a more accurate explicit straining point location procedure for the final approximation predicted result, in contrast to the usual implicit procedure employed in all the previous realizations (References 1-7, 11, 12, 15) of the method.

This feature considerably enhances the capability and generality of the method. The explicit straining procedure, which in essence specifies the points at which the final solution results are determined rather than allow these points to be determined implicitly from the straining of the base flow points as was done standardly in the past, avoids a double interpolation of the approximation result. In a series of tests on highly-supercritical airfoils, this new procedure has been found to yield significantly improved accuracy in high-gradient regions at only a very slight increase in computational work.

One of the most important results to emerge from the calculations involved in these case studies was the discovery of a particular deficiency of the approximation method, and the subsequent development of the means to improve the accuracy of the approximation predictions in shock regions and other high gradient regions. The improvement in the basic procedure developed to meet these requirements consists of employing additional invariant points in those high gradient locations. For example, it was found that by characterizing a shock which has a post-shock expansion region, as sketched below,



with five invariant points -- which correspond to: (1) pre-shock minimum pressure, (2) maximum gradient point, (3) post-shock maximum pressure, (4) post-shock minimum expansion pressure location, and (5) point of inflection between points (3) and (4) -- rather than just the one point corresponding to the critical pressure location, which was standardly done in the past; that significantly improved approximation results are obtained in the shock region for extreme solution extrapolations. The primary difficulty of this five point characterization of the shock is that at this time it is not automatically implemented in the present method, but requires user intervention. Consequently, this more accurate procedure was not employed in the optimization studies reported below. In future work this improvement should be automated and included in the method.

3.2 PRELIMINARY APPLICATION OF APPROXIMATION METHOD TO SUPERCRITICAL WING OPTIMIZATION

The ultimate general utility of the approximation methods developed and evaluated here lies in their application to problems involving the repetitive use of computationally-expensive codes to determine a large number of related nonlinear flow solutions. The concept is that these approximation methods would be coupled with these high run-time computational codes and used in conjunction with them in those applications to reduce the computational requirements substantially. This would be accomplished by employing the approximation method to reduce to a certain minimum the number of the computationally-expensive solutions required to accomplish the application.

The specific application of the approximation concept in this study is toward the general supercritical wing design optimization problem. One of the major objectives of the present investigation was the demonstration that the approximation method is capable of working effectively in such an important and practical nonlinear design environment. Toward that objective, the approximation procedure was implemented as follows. First, the approximation method was configured in a form suitable for prediction of surface properties on three-dimensional supercritical wings. Next, that method was then integrated with a proven wing design optimization procedure capable of treating supercritical wings. The optimization procedure selected was the FLO22OPT method recently developed at Ames Research Center. That procedure consists of the CONMIN optimization driver (Ref. 8) coupled with the FLO22 full potential three-dimensional flow solver (Ref. 9). The latter method is capable of determining transonic flows past either wing alone or wing-body combinations. Finally, applications were made of the combined procedure to several case studies involving wing section profile optimization. The objectives of these initial applications were to demonstrate the workability of the approximation concept in a supercritical wing design environment, provide a benchmark of the potential for computational savings of the combined approximation/optimization procedure for some typical design problems, determine the accuracy of the approximation-predicted results for these cases, and finally identify any areas of improvement required for more general wing optimization studies.

The particular wing design optimization problems selected for study involved the alteration of a baseline sectional profile shape at various spanwise locations along the wing by adding to that baseline profile a set of shape functions according to the relation

$$Z(x) = Z_0(x) + \sum_{i=1}^M (DV_i - 1) F_i(x) \quad (23)$$

where Z_0 are the ordinates of the baseline profiles, F_i are the shape functions, and the coefficients DV_i are the design variables. Those

values are determined by the optimization program as a result of a search through design-variable solution space to achieve a desired design improvement. Here for convenience we have chosen the coefficients of F_i to be (DV_i-1) rather than DV_i . The general class of geometric shape functions employed here, and which have been found to be successful in previous applications involving optimization of supercritical airfoil sections (Ref. 16), consists of exponential decay functions and sinusoidal functions. These are of the general form $(1-x) \cdot x^p / e^{qx}$ and $\sin(\pi x^r)^n$, where the exponents p , q , r , and n are selected to provide a desired maximum at a particular chordwise location. The exponential functions are generally employed to provide adjustments near the leading edge, while the sinusoidal functions are used to provide maximum ordinate changes at particular chordwise stations. Illustrations of the chordwise variation of typical members of these classes of shape functions are provided in Figure 6, and it can be seen that these functions smoothly concentrate ordinate thickness at selected locations.

A strategy which has proved convenient for performing optimization studies involving aerodynamic performance parameters (Ref. 16) has been to recontour the profile shape so as to tailor the surface pressure distribution to conform to a desired distribution. This type of objective provides local control over the basic aerodynamic surface flow property of importance, and provides a means of attempting to achieve aft pressure gradients sufficiently weak to avoid separation. An important corollary advantage of using such an objective is that viscous separation can be minimized. This allows use of an inviscid aerodynamic flow solver in the optimization process rather than a much more computationally-expensive viscous solver, and assures that the optimization result thus obtained at the inviscid level is representative of the actual flow.

In such studies, the characteristics that are primarily sought after in the optimization process are the minimization of both the peaky behavior near the leading edge and the compressive gradient on the aft portion of the suction surface that typically exist on the baseline profiles considered. This is illustrated schematically in Figure 7.

Consequently, for these preliminary case studies the overall performance objective was, through modification of the surface contour, to tailor the pressure distribution along a portion of the upper surface so as to conform to a desired distribution. The objective function was taken as the minimization of the mean squared error between the predicted and desired surface pressure distribution, i.e.,

$$OBJ = \sum_{K=1}^K [C_{p_{predicted}}(x_K) - C_{p_{desired}}(x_K)]^2 \quad (24)$$

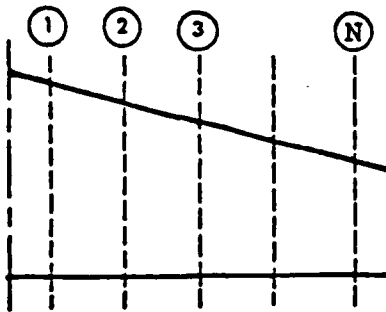
where K represents the number of chordwise locations x_K where desired and calculated surface pressures are compared.

In developing the combined APPROX/FL0220PT program, we have incorporated several features that enhance the robustness and user use of employing the code. These ideas were based on our experience with the approximation method in two-dimensional optimization applications (Ref. 7) for nonlinear internal flows. One of these features was the development and use of the explicit straining procedure mentioned previously. The most important of the other features relates to the way that the calibration solution matrix required by the approximation method is defined. Two separate options have been incorporated in the current program. The first option employs the approximation method in its usual fashion whereby the user specifies the matrix of calibration flow solutions to be determined. These solutions correspond to nearby (to the baseflow) solutions associated with the separate variation of each of the M design variables appropriate to the optimization problem under consideration. With this option, calibration matrix definition can be accomplished by either individually specifying all the design variables or alternatively by employing a constant-value calibration step size which increments each design variable by a fractional change of its base flow value.

The second of these options provides the user with a basically automatic hands-off procedure for using the perturbation method. Under this option, the user is not required to preselect and input the design variable values for the calibration solution matrix. Rather the matrix is determined completely by the program in the following way. For the first optimization cycle, the approximation method is not used. Full nonlinear aerodynamic solutions are determined by the flow field code as required as input for the gradient and search optimization calculations. After the first search cycle is complete and a new design point determined, design variable values for the calibration solution matrix are then determined based on the direction that the first search cycle has taken. We have in series of tests involving two-dimensional airfoil optimization studies verified that such a procedure results in an extremely good definition of the calibration solution matrix. The result is that the design variable solution space which is subsequently searched on the second and successive optimization cycles usually requires only very reasonable interpolations/extrapolations within the design variable parameter range of the defined solution matrix. Consequently, this option provides an automatic, user-invisible procedure for defining the calibration solution matrix. It does, however, require the additional computational cost of one optimization search cycle using the exact nonlinear aerodynamic flow solver in contrast to the first option which requires the user input of the design variable values for the calibration solutions. Nevertheless, this second option provides a highly accurate and basically hands-off means of employing the approximation method, and we recommended it for use when little or no information is available on search direction from previous or related calculations.

The wing design problem that the combined APPROX/FL0220PT program has been initially directed toward is the performance of wing section profile design optimization at a specified number N of selected spanwise

stations along the wing. As shown below, this would proceed sequentially one station at a time from root to tip.



In this procedure the effect of profile changes of outboard stations on previously-optimized inboard stations is initially ignored, but subsequently checked after the final outboard station is optimized. If the optimization changes on the outboard stations have resulted in a degradation of the performance of the inboard stations, then the procedure can be repeated with another inboard to outboard sweep using the final design variable information obtained from the previous pass. Previous experience has shown that for wings swept at approximately 35° or more, a single optimization pass is sufficient. Wings of lower sweep may require an additional pass.

For the initial application of the combined approximation/optimization method, we have tested the complete procedure on a five design variable problem involving supercritical surface pressure tailoring for transonic flows at $M_\infty = 0.85$ and $\alpha = 1^\circ$ past the wing described previously in Figure 3. We employed the following shape functions

$$Z(x) = Z_{\text{base}}(x) + \sum_{i=1}^5 (DV_i - 1) \sin(\pi x^{r_i})^3 \quad (25)$$

with the base profile shapes of the wing sections taken as a NACA 65A215 profile and the exponents $r_i = (0.301, 0.431, 0.576, 0.756, 1.000)$. These particular sinusoidal shape functions achieve their maxima at (10%, 20%, 30%, 40%, 50%) of local chord. The desired pressure distribution chosen as the target was taken simply to be

$$C_{p_{\text{desired}}}(x/c) = -0.10 \quad , \quad 0.05 \leq x/c \leq 0.50 \quad (26)$$

at the selected spanwise stations to be optimized.

In Figures 8 and 9 we demonstrate, in terms of final design variable accuracy and potential computational time savings, comparisons of results between the approximation method and those obtained when using the exact nonlinear solutions for this 5 design variable supercritical wing surface pressure tailoring case study. The optimization was performed at the root chord station using the program option to automatically define the calibration solution matrix. Figure 8 provides a comparison of the approximation predicted final design variables and objective function (●) with those obtained with not using the approximation method but employing exact nonlinear full potential aerodynamic solutions throughout (○). These are the results after 5 optimization cycles. We note the essentially exact correspondence between the final design variable values. Corresponding comparison of the objective function shown on the right, in fact, indicates a slightly better result obtained by the approximation method (■). In Figure 9, the corresponding comparison of computational work in CPU seconds and objection function reduction per optimization search cycle is provided. Here we see that after the first cycle is complete, the approximation procedure actually requires less time to define the calibration matrix solution and complete the second search than does the full nonlinear flow field method with the same reduction in objective. From that point on, the approximation method requires essentially no time to complete searches 3 to 5, and then an additional increment to calculate the final design result using the nonlinear flow field solver. Time savings achieved with the approximation method for this case is 58% of that required for the exact nonlinear result.

Figures 10 and 11 provide similar results for the spanwise station $y/s = 0.52$. Again we observe that the approximation procedure is able to provide very accurate results in comparison to the results obtained when not employing the approximation method but using the nonlinear flow solver throughout the optimization search. Time savings for this case was 55% of that required for the exact nonlinear result.

In Figures 12-15, we have provided analogous comparative results of the approximation method obtained when employing the option of specifying the calibration solution matrix. In this case study, we have used a uniform design variable stepsize increment of $DV_i = -0.001$ for each of the five design variables. This represents a better than usual estimate (see Figures 8 and 10) of the calibration matrix in the sense that only modest interpolations and extrapolations will be required of the approximation method by the optimization procedure as it searches through design solution space for the optimum. Without some a priori knowledge of the direction of the optimization search, large interpolations and extrapolations are often required of the approximation method. This usually results in a subsequent degradation in accuracy of the approximation predictions under this option. For this case study, as might be anticipated from the above discussion, the approximation-predicted design variables are in as good agreement with the exact nonlinear result, as they were in the previous comparisons which employed the program option to automatically define the calibration solution matrix. The computational time savings in this instance are shown in Figures 13 and 15, and as expected are greater

(68% vs. 58% at $y/s = 0.0$; 64% vs. 55% at $y/s = 0.52$) than before due to the savings of essentially one optimization cycle involving the use of the exact nonlinear flow solver.

A series of related optimization studies analogous to those presented in Figures 9-15 was next undertaken. A common problem quickly became apparent in these studies that led to inaccuracies in the approximation-predicted results. This problem was related to the sudden occurrence, during the optimization process, of additional shocks. For example, this could be a high strength shock appearing near the wing leading edge, often forming initially at the wing tip and working inboard, or it could be the formation of an additional shock near the wing midchord line that would intersect the main shock. Several of these more complex shock topologies that occurred are illustrated in the bottom of Figure 16, together with the more usual shock patterns that were observed. Although the approximation method is capable of simultaneously treating multiple shock and high gradient regions, a fundamental limitation of the method as it is presently constituted is that the basic topology of the solutions that it is approximating must remain the same. This means that, over the range of parameter variation that the method is to be employed, no shocks or other high-gradient regions be either added or lost. This limitation appears not to be satisfied for many of the supercritical wing optimization problems of interest, in which an additional shock can suddenly appear during the optimization process, but disappear or weaken considerably as the optimized design point is approached. Our experience in two-dimensional supercritical flow optimization problems has been that this phenomena almost never occurs. It appears in the three-dimensional problem that the additional degree of freedom that the flow has allows these sudden occurrences to happen when sensitive supercritical flows are simultaneously perturbed in various ways. The remedy to this is to extend the basic capability of the approximation method so that such topology changes can be allowed. This is feasible, but will require an extension of the method in which additional calibration solutions are employed that allow the approximation solution to cover more than one topology. Generally speaking, one additional calibration solution matrix will be required for each topology change. Development of the approximation method, in a fashion that can automatically sense and extend itself to cover topology changes, appears to be a necessary refinement in order to extend the utility of the approximation method to general three-dimensional supercritical wing optimization design problems.

4. CONCLUSIONS AND RECOMENDATIONS

An investigation was conducted involving the preliminary development of approximation procedures for determining rapid and accurate approximations to three-dimensional transonic flows past supercritical wings. The ultimate purpose is to establish a method for minimizing computational requirements associated with optimization design studies of supercritical wings. The procedures being developed employ unit perturbations, determined from two or more nonlinear 'base' solutions which differ from one another by a nominal change in some geometry or flow parameter, to predict a family of related nonlinear solutions which can be either continuous or discontinuous. The results reported here relate to the extension of the previously-developed successful method for multiple-parameter two-dimensional perturbations: (1) to simultaneous multiple-parameter three-dimensional perturbations, and (2) to the preliminary application of the multiple-parameter three-dimensional procedure in combination with the FL022OPT optimization procedure to some initial design problems.

Calculations based on three-dimensional full-potential nonlinear solutions have been carried out to establish the accuracy and range of validity of the multiple-parameter three-dimensional capability. These involved flows past transport aircraft wings involving simultaneous changes in both flow and geometric parameters, with attention focused on strongly supercritical situations involving large surface shock movements over the parameter range studied. Preliminary applications of the multiple-parameter perturbation method coupled with the FL022OPT optimization procedure were made to wing design problems in order to examine the capability of the method to produce accurate results in a typical design environment, and also to evaluate its potential for computational savings. This was focused primarily on supercritical case studies involving multiple-design variables. Sensitivity studies were also performed to examine the accuracy dependence of the perturbation method on the choice of the initial calibration solution matrix.

Comparisons of the multiple-parameter perturbation results with the corresponding 'exact' nonlinear solutions display good accuracy and indicate a large range of validity for those situations where the basic flow topology does not change over the parametric range studied. This is in direct correspondence with previous results for strongly supercritical two-dimensional flows. The preliminary case studies of the multiple-parameter perturbation method combined with optimization procedures have clearly demonstrated the capability of the method to work accurately in a design environment, and have also established the methods' potential for reducing the computational work required in such applications by factors approaching an order of magnitude. Sensitivity studies indicate that for supercritical flows, although the choice of the initial calibration matrix is important, the approximation predictions can nevertheless maintain reasonable accuracy even for poorly-selected calibration matrices. Because general supercritical

wing optimization problems often involve changing shock and/or high-gradient region topologies, it appears that an extension of the present method is appropriate and needed to treat this more general problem.

Based on these results, we conclude that approximation methods formulated on these ideas are both accurate and clearly workable in design environments, and can provide the means for substantially reducing the computational work required in such applications. We suggest the development of the combined multiple-parameter approximation procedure and optimization methods into a robust design tool for supercritical wing design.

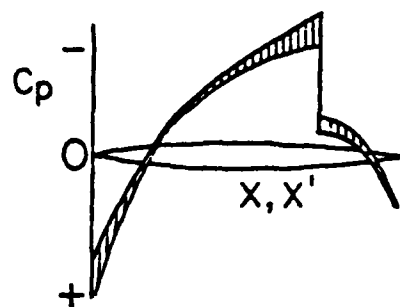
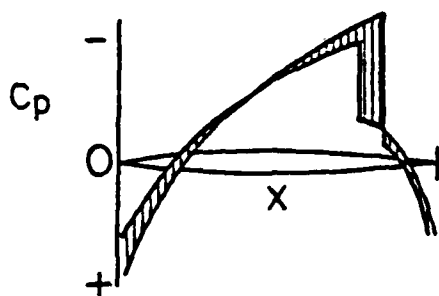
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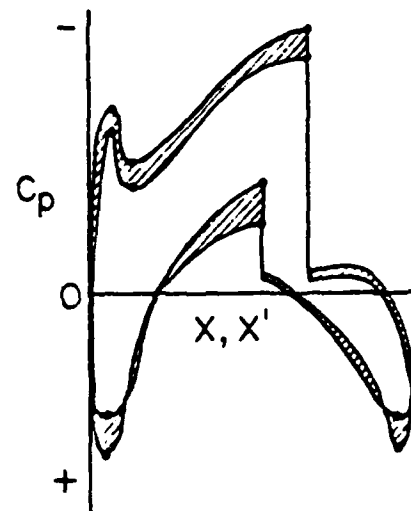
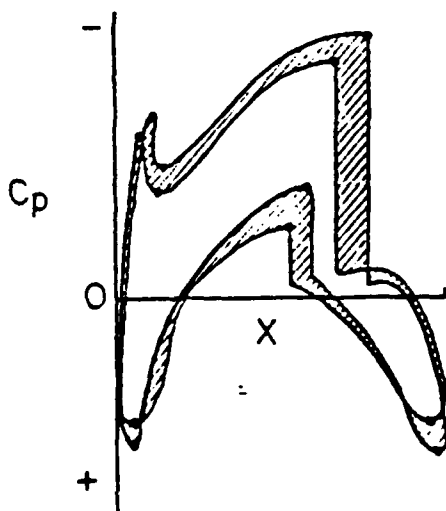
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Perturbation for
calibration solution
in physical coordinates

Perturbation for
calibration solution
in strained coordinates



(a) Single shock.



(b) Multiple shock and high-gradient locations.

Figure 1.- Illustration of the coordinate straining concept for the application of perturbation methods to nonlinear flow problems; shaded areas denote the perturbation between two nonlinear, discontinuous, transonic flows with (right hand plots) and without (left-hand plots) coordinate straining

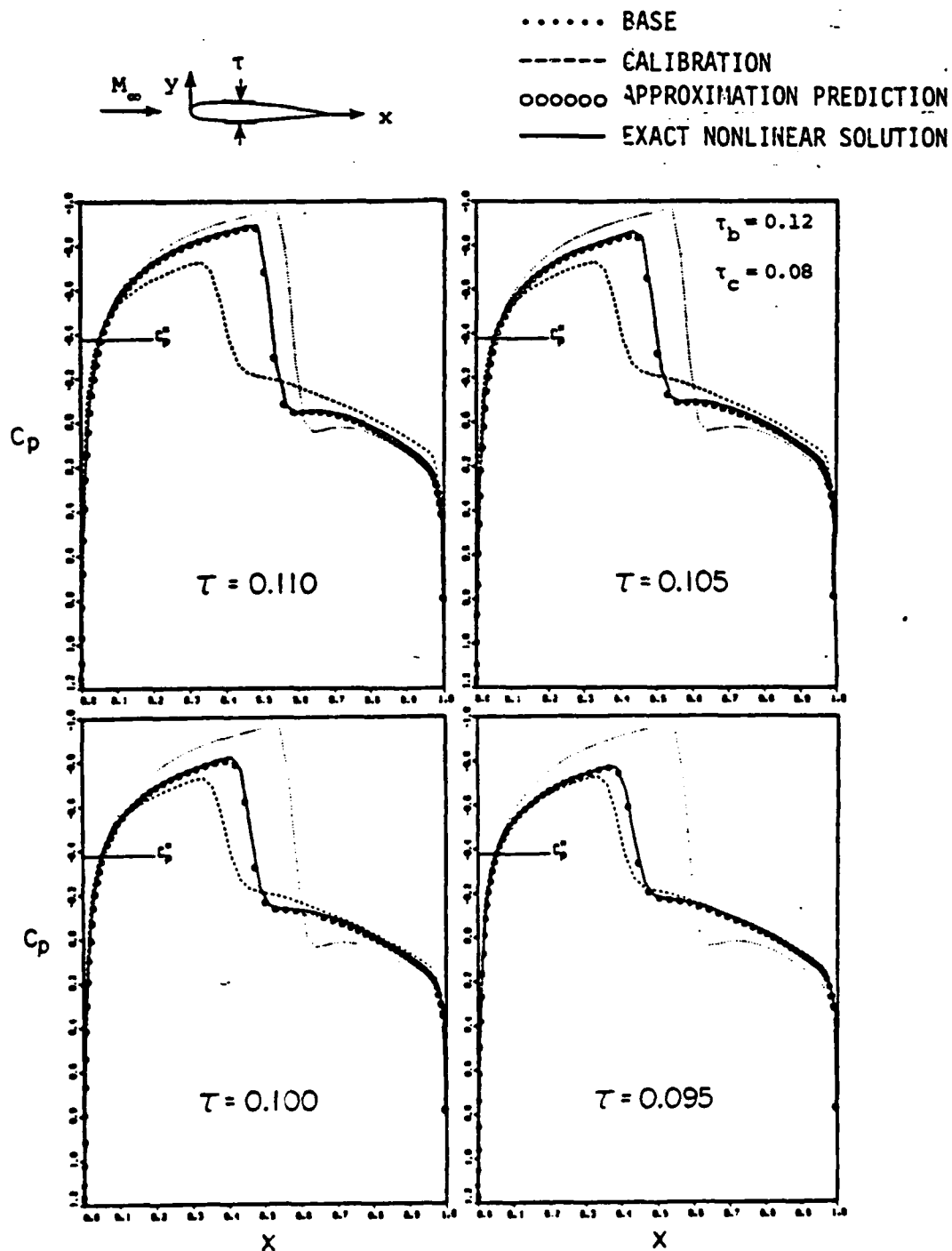


Figure 2. - Comparison of approximation-predicted (o) and exact non-linear (—) surface pressures for thickness-ratio perturbations of strongly supercritical transonic flows involving isolated NACA 00XX airfoils at $M_\infty = 0.820$ and $\alpha = 0^\circ$.

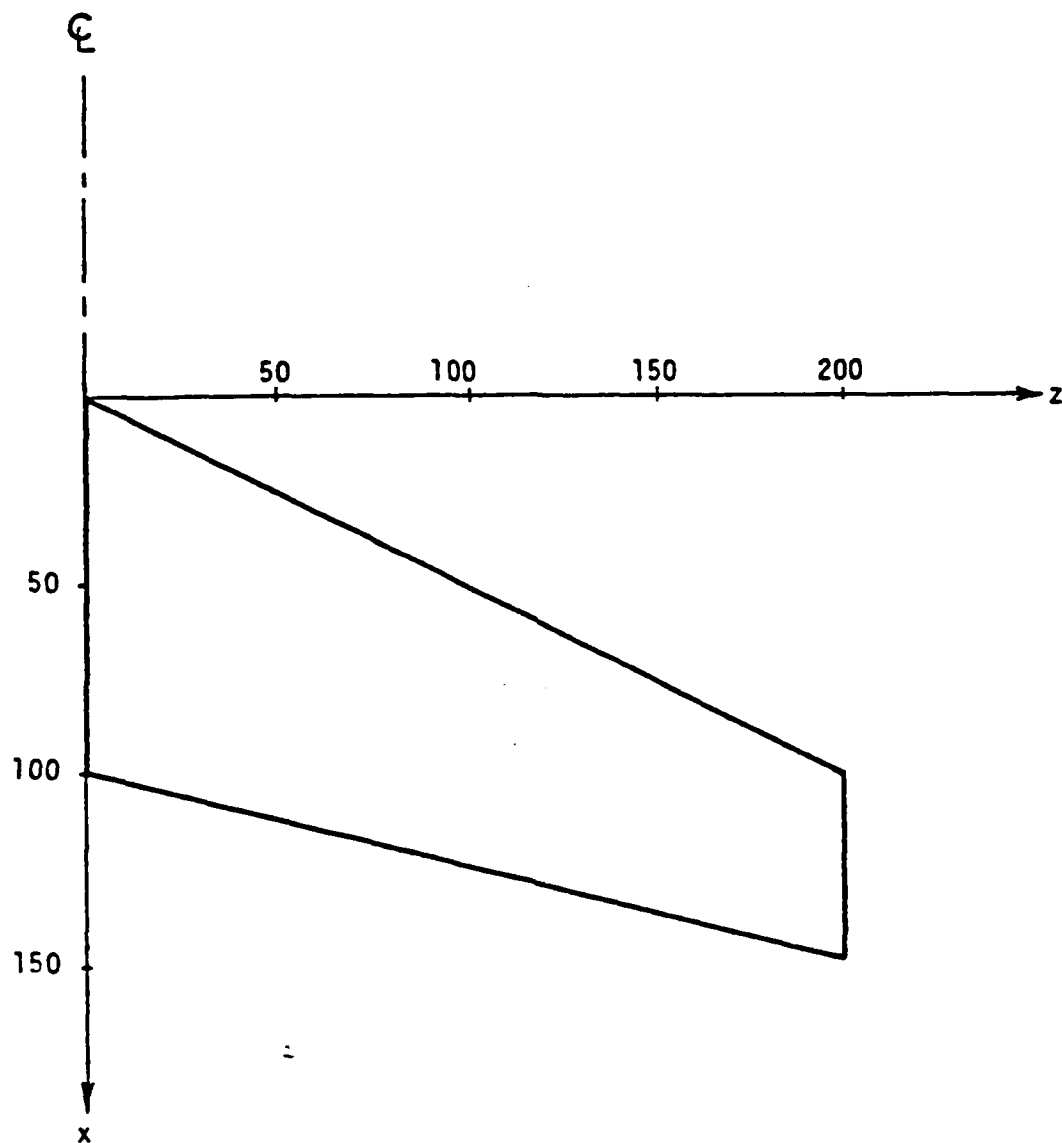


Figure 3. - Planform of wing employed in supercritical case studies or approximation method.

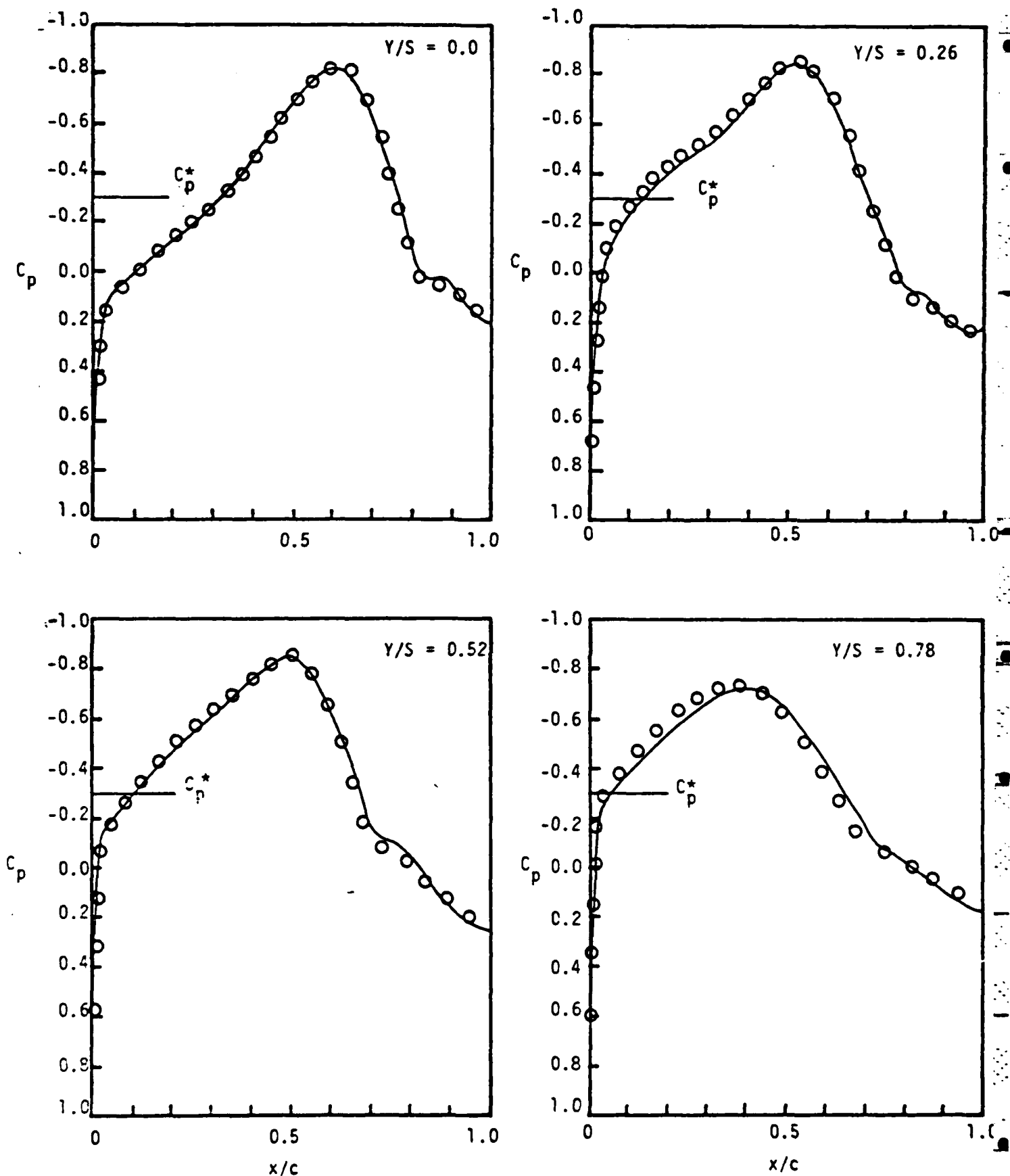


Figure 4. - Comparison of approximation method (O) and exact nonlinear (-) surface pressure results for the simultaneous two-parameter perturbation of (M_∞, α) for a strongly supercritical wing flow.

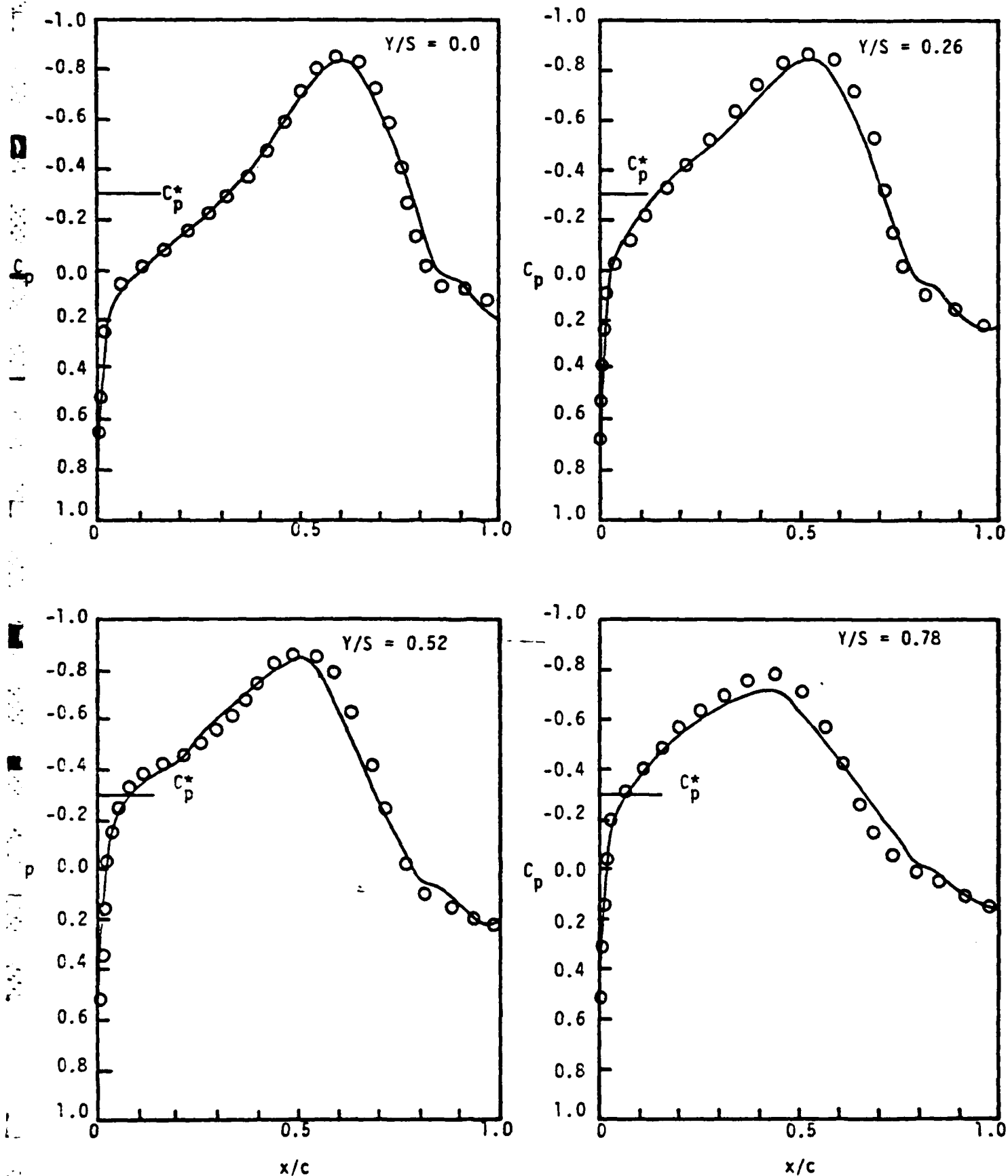


Figure 5. - Comparison of approximation method (O) and exact nonlinear (-) surface pressure results for the simultaneous three-parameter perturbation of (M_∞, α, τ) for a strongly supercritical wing flow.

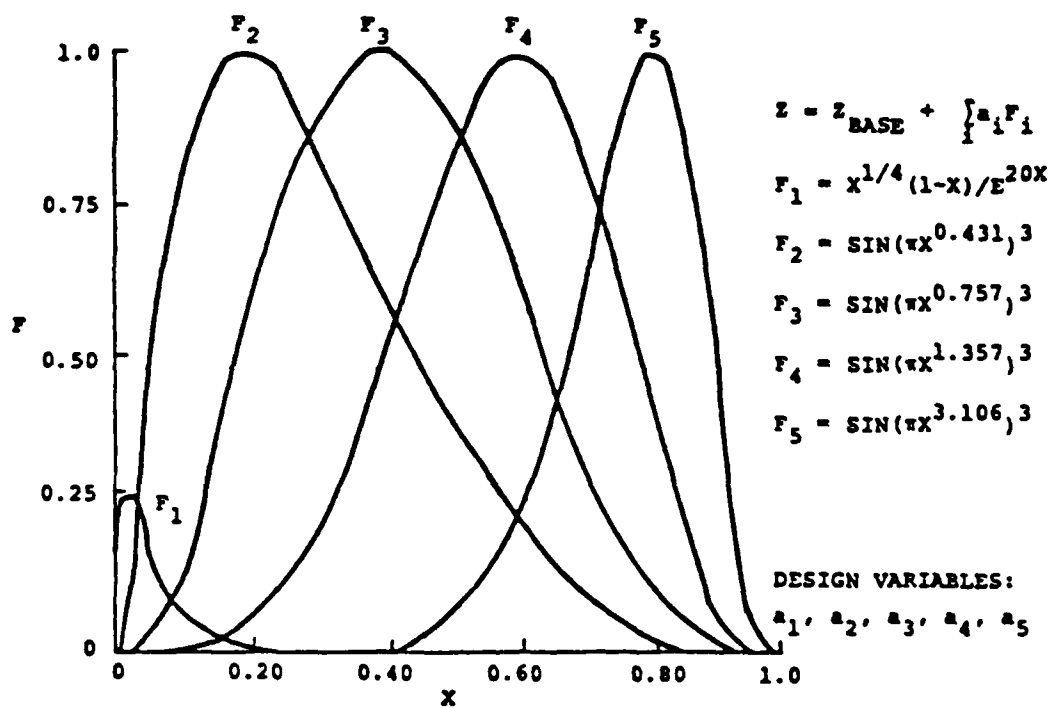


Figure 6. Illustration of typical ordinate shape functions F_i employed in blade contour alteration optimization problems.

N OBJECTIVE IS TO MINIMIZE:

$$\sum_{i=1}^N (C_p(x_i) - C_p(x_i)^{\text{Desired}})^2$$

Extent of airfoil
surface contoured

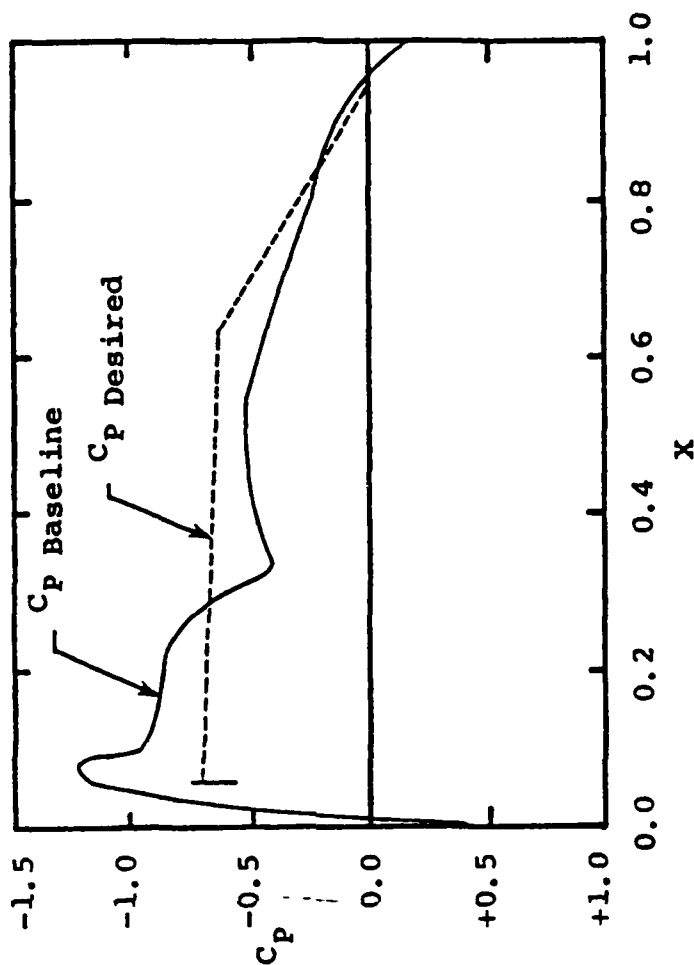


Figure 7.- Illustration of physical basis of optimization problem involving profile surface contouring to tailor the surface pressure distribution to a desired distribution.

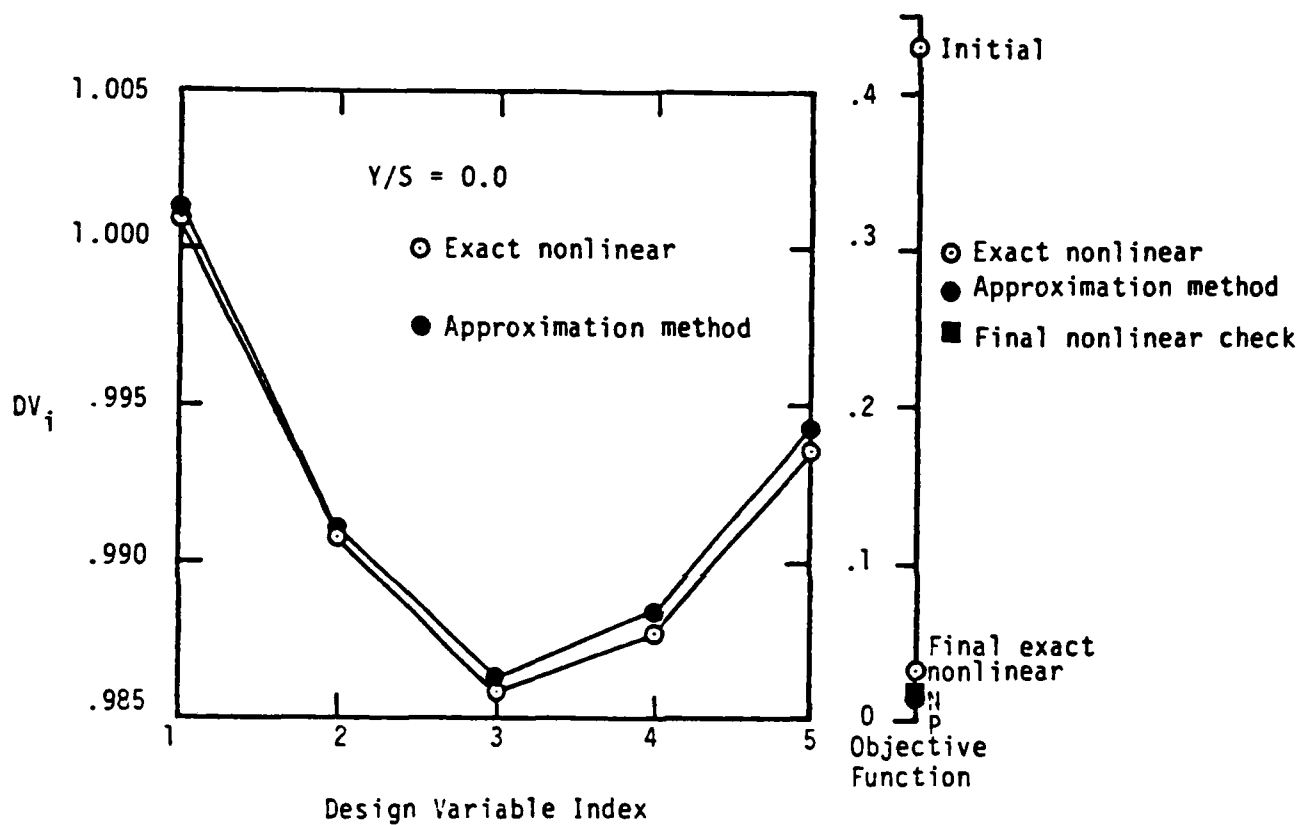


Figure 8. - Comparison of approximation predicted and exact nonlinear results for final design variables and objective function for 5 designvariable supercritical case study with surface pressure tailoring objective at the wing root chord station.

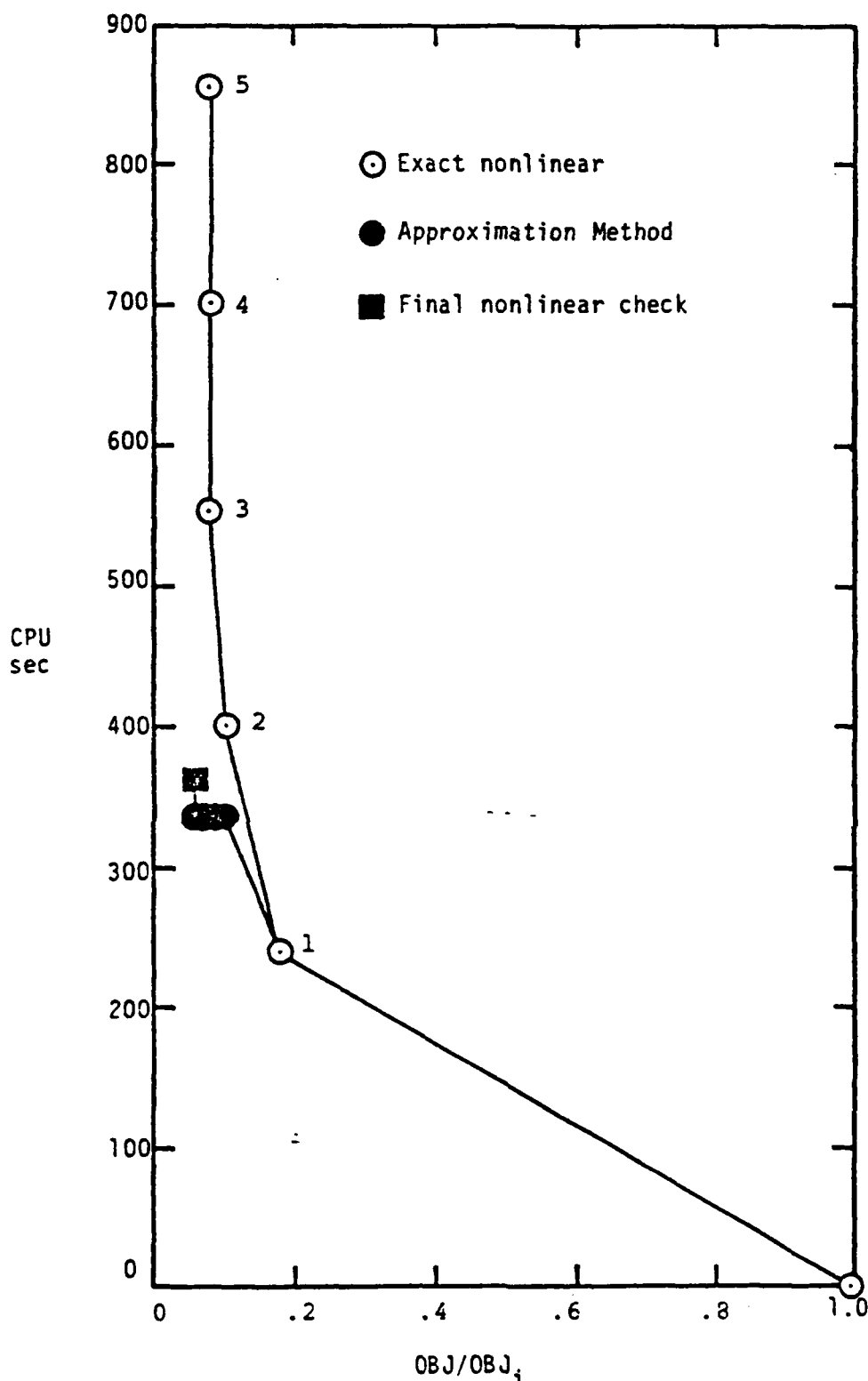


Figure 9. - Comparison of computational work and objective function reduction per optimization search cycle when employing approximation method after first search cycle (●) or when using exact nonlinear solutions (○) for 5 design variable supercritical case study with surface pressure tailoring objective at wing root chord.

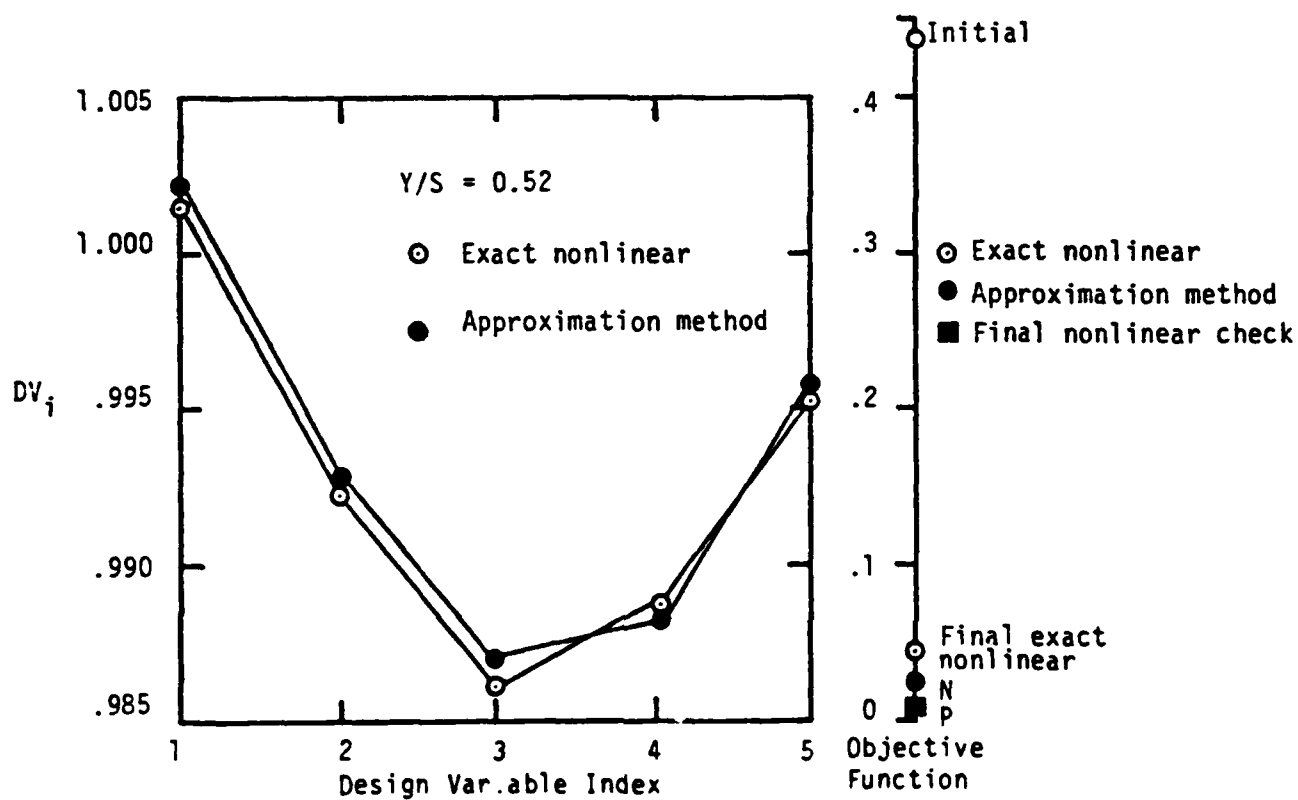


Figure 10. - Comparison of approximation predicted and exact nonlinear results for final design variables and objective function for 5 design variable supercritical case study with surface pressure tailoring objective at the $Y/S = 0.52$ spanwise location.

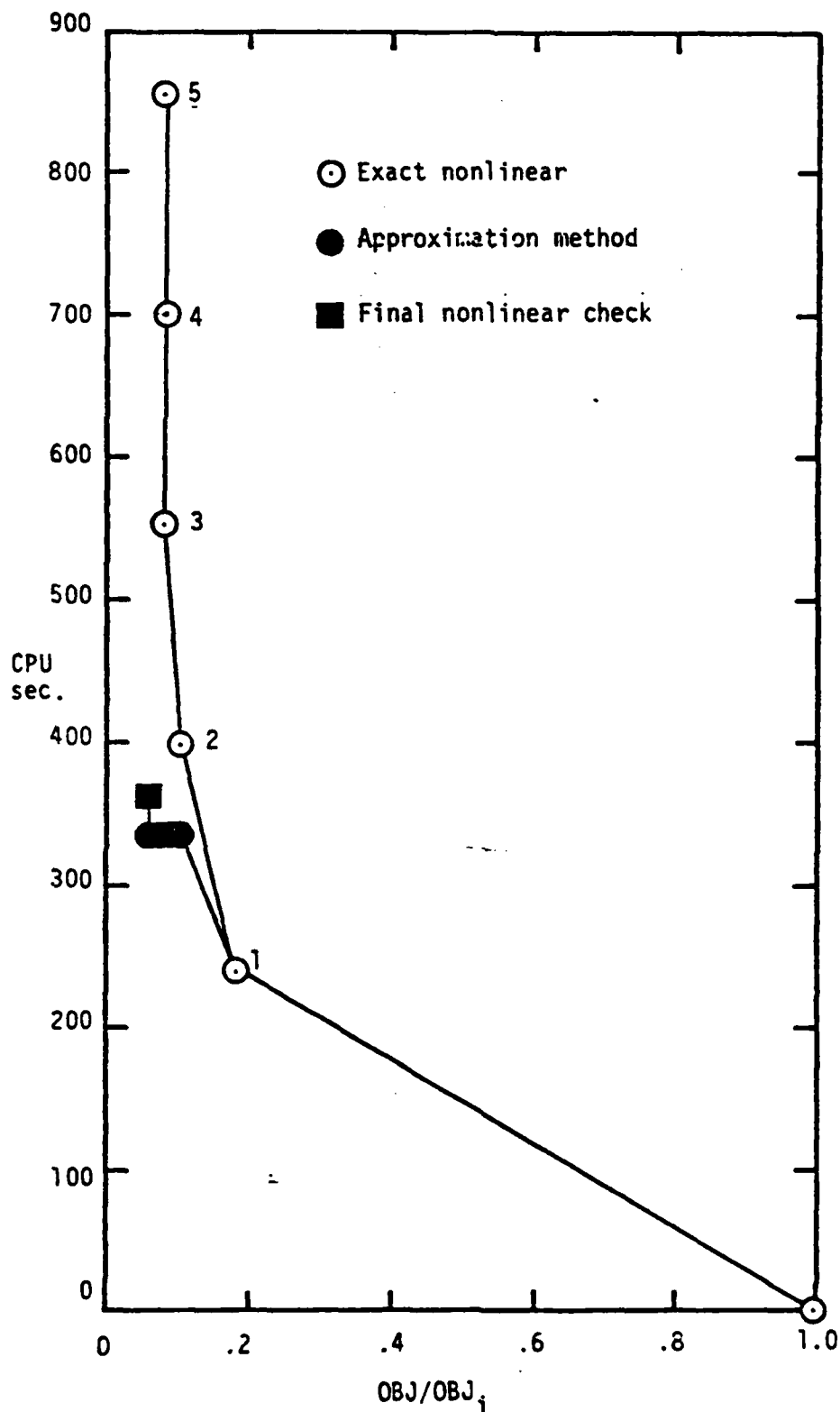


Figure 11. - Comparison of computational work and objective function reduction per optimization search cycle when employing approximation method after first search cycle (●) or when using exact nonlinear solutions (○) for 5 design variable supercritical case study with surface pressure tailoring objective at the $Y/S = 0.52$ spanwise location.

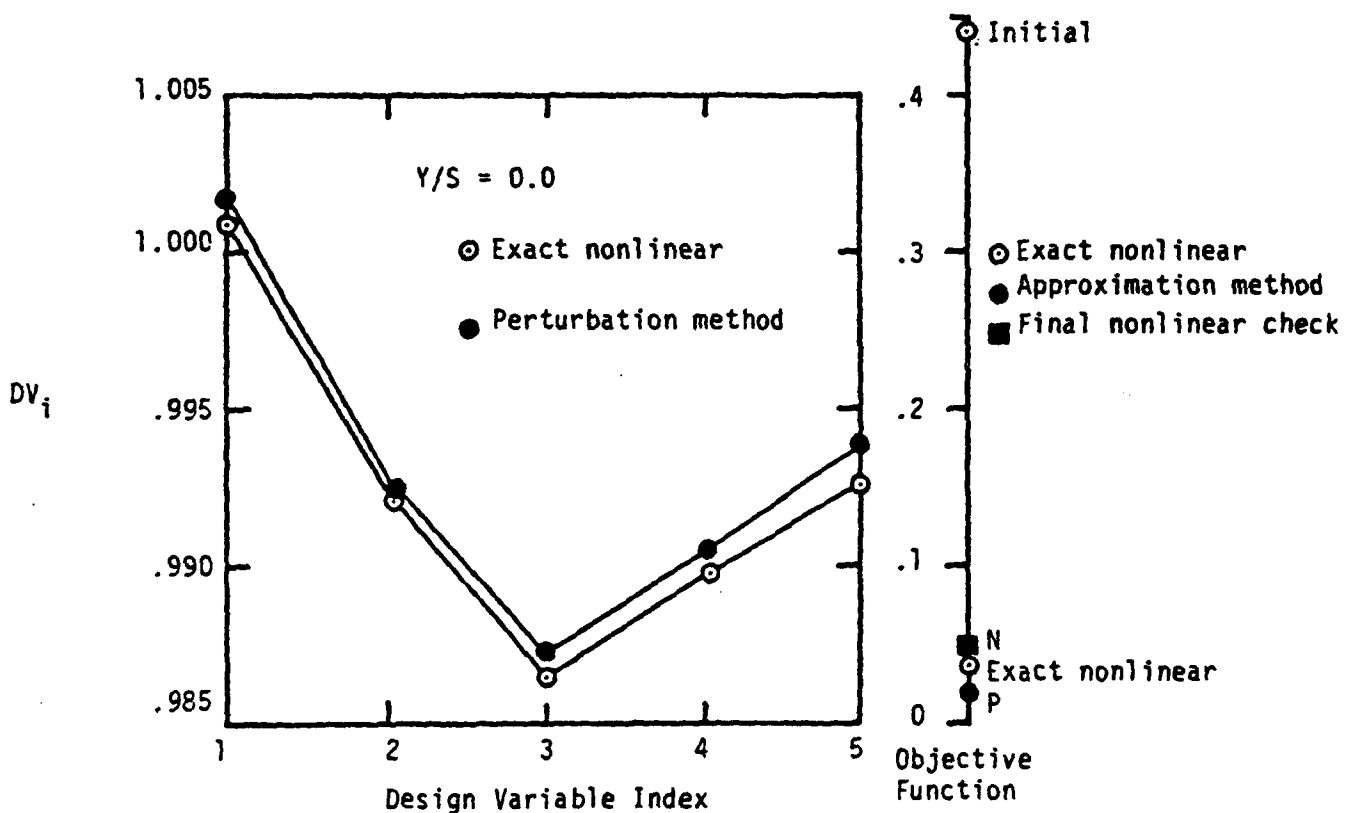


Figure 12. - Comparison of approximation predicted and exact nonlinear results for final design variables and objective function for 5 design variable supercritical case study with surface pressure tailoring objective at the wing root chord station.

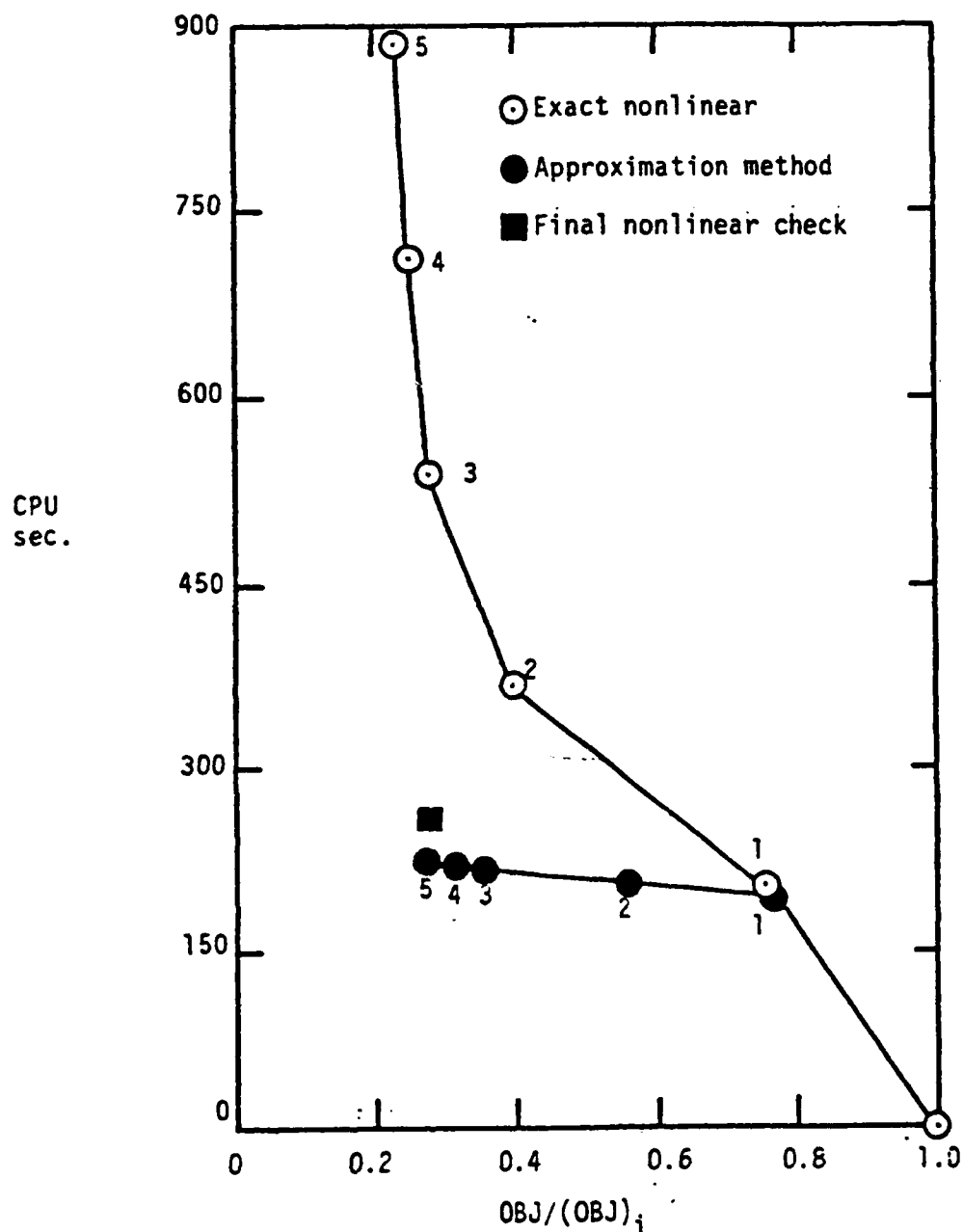


Figure 13. - Comparison of computational work and objective function reduction per optimization search cycle when employing approximation method (●) or exact nonlinear solutions (○) for five design variable supercritical optimization case study with surface pressure tailoring objective at wing root chord.

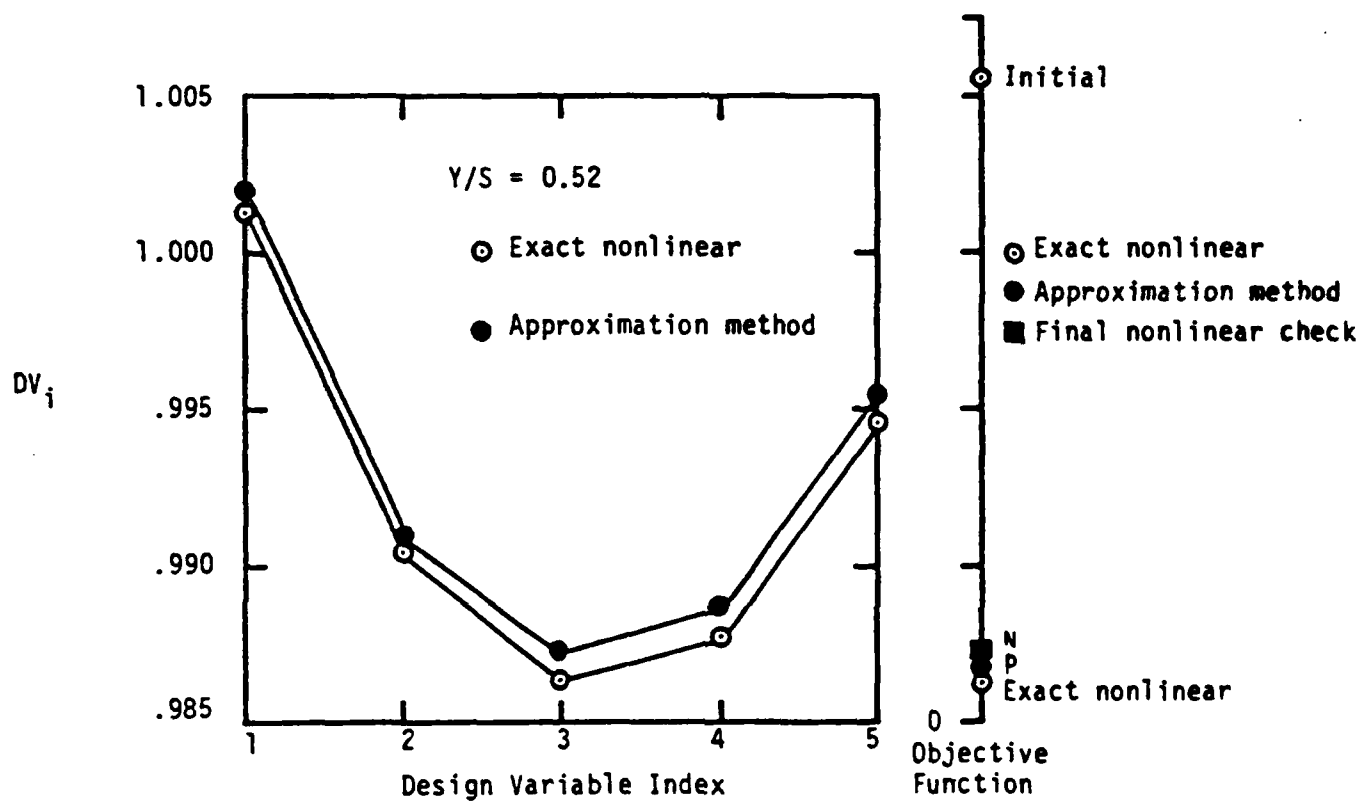


Figure 14. - Comparison of approximation predicted and exact nonlinear results for final design variables and objective function for 5 design variable supercritical case study with surface pressure tailoring objective.

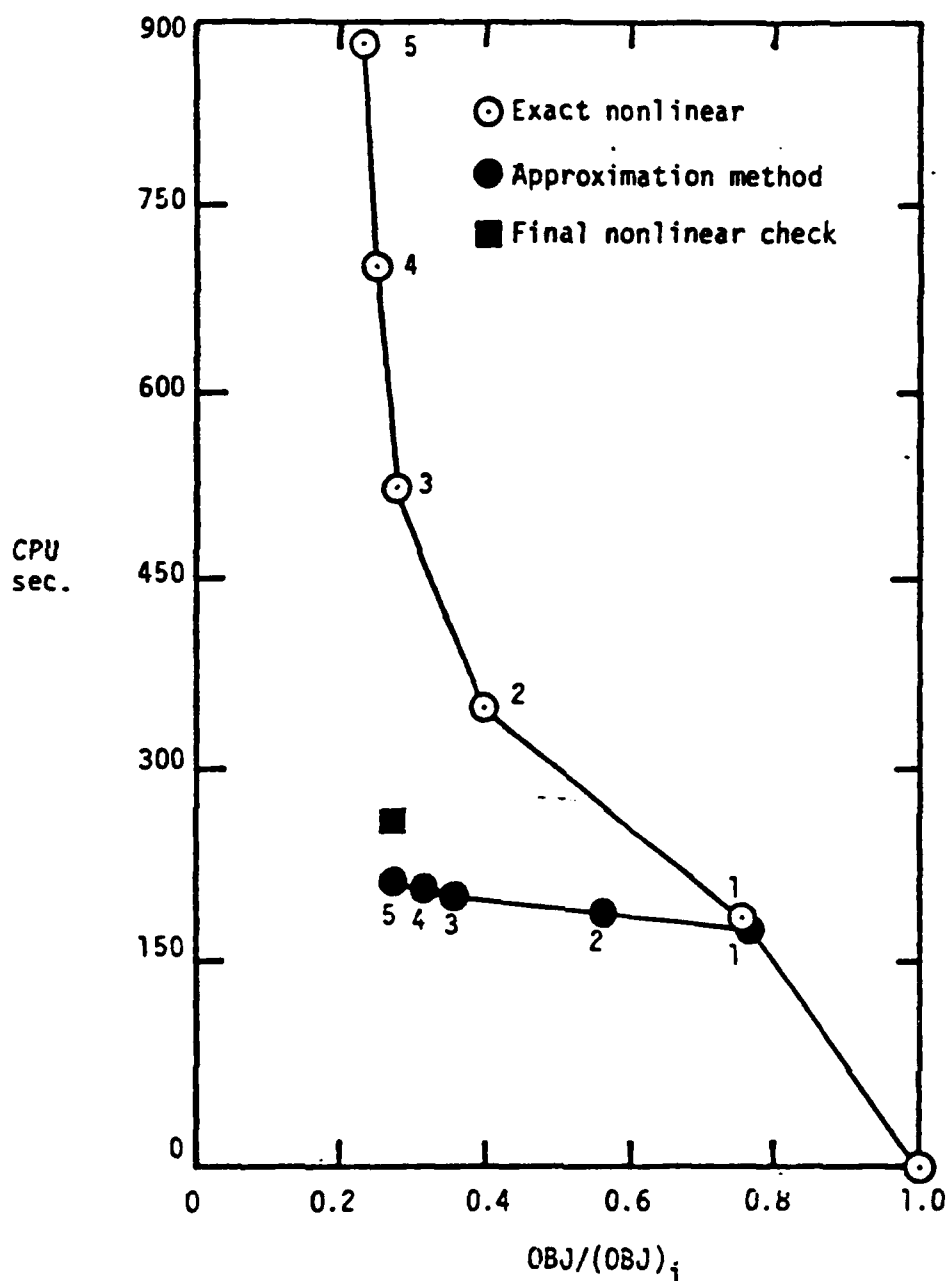
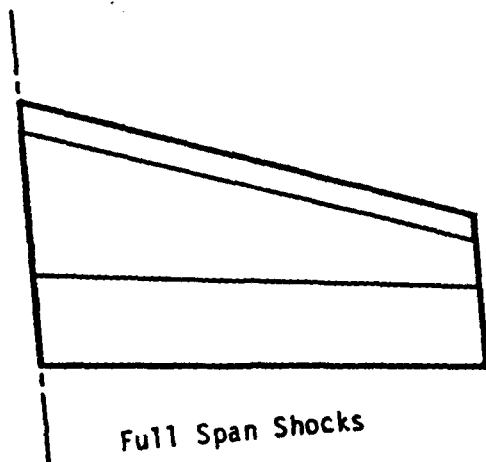
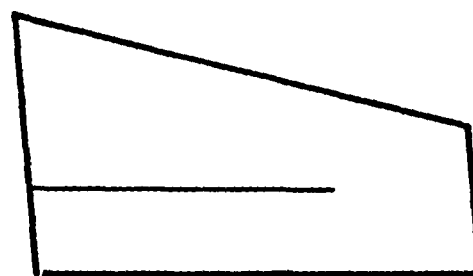


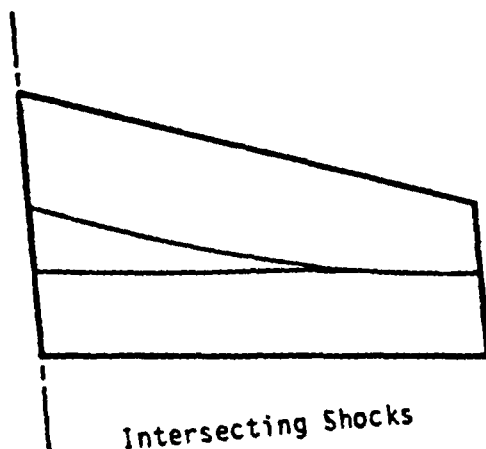
Figure 15. - Comparison of computational work and objective function reduction per optimization search cycle when employing approximation method (●) or exact nonlinear solutions (○) for five design variable supercritical optimization case study using a surface pressure tailoring objective at the Y/S = 0.52 spanwise location.



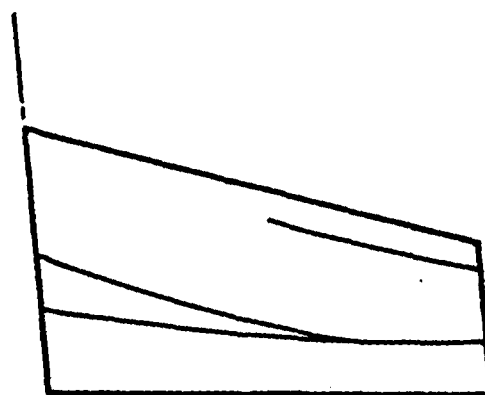
Full Span Shocks



Partial Span Shock



Intersecting Shocks



Partial Span and Intersecting Shocks

Figure 16. - Illustration of several shock topologies possible for supercritical wing flows

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